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Math on a Sphere

Topic 11: Angles on a Sphere

Until now we've used these Topic pages to discuss commands or syntax for the Math on a Sphere programming language. In this Topic we're going to take a different approach and discuss a strange phenomenon you may have noticed.

You may have noticed that angles on a sphere are weird. And not just a little weird – really, really weird. As in, they violate all the normal rules we're used to. Think this is wrong? Just check out this example:

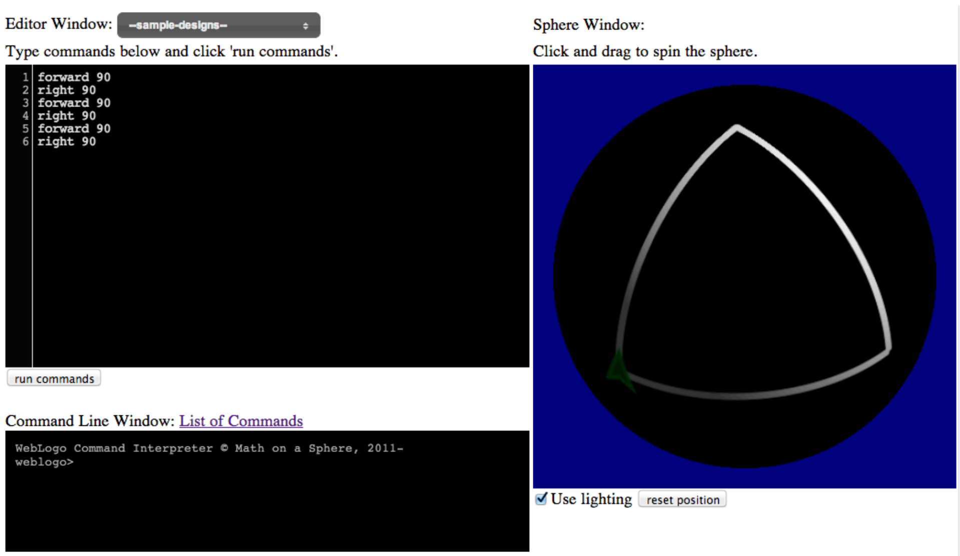

Figure 1. A large triangle drawn on the sphere

Here is the code that made this example:

```
forward 90
right 90
forward 90
right 90
forward 90
right 90
```

What do you notice?

We just made a **triangle** with **three 90 degree angles**! How can that be?

Because the surface of a sphere is curved, when we draw shapes on a sphere they do not follow the usual geometry rules we are familiar with (which assume Euclidian geometry). And it's not just triangles that have weird angles. Any shapes that we draw on the sphere can have weird angles.

One way to try to understand the weird world of spherical geometry is to try drawing some shapes on a sphere, such as an orange or a ball. After you've drawn a shape, use a protractor to measure the angles. Do you see any kind of pattern? How do the angles compare when you make draw small triangles versus large ones?

If you're interested in learning more about this, we recommend the following websites:

- Spherical trigonometry (Wikipedia)
  http://en.wikipedia.org/wiki/Spherical_trigonometry
- Spherical excess (Wolfram)
  http://mathworld.wolfram.com/SphericalExcess.html
- A primer on spherical geometry and trigonometry
  http://www.astro.sunysb.edu/fwalter/AST443/sphgeo.html
- Spherical trigonometry
  http://www.math2earth.oriw.eu/publications/15_Spherical%20trigonometry.pdf

Angles on a sphere may be confusing, but they let us make awesome designs. One of our favorite designs is a shape-changing spiral. Before we get to the shape-changing spiral, let's see how we can make a "normal" spiral. The following code shows one way we can create a spiral:

```
x = 20
a = 20

repeat 60 {
    forward x
    right a
    x = x - 0.33
    a = a + 0.5
}


penup
forward 50
```

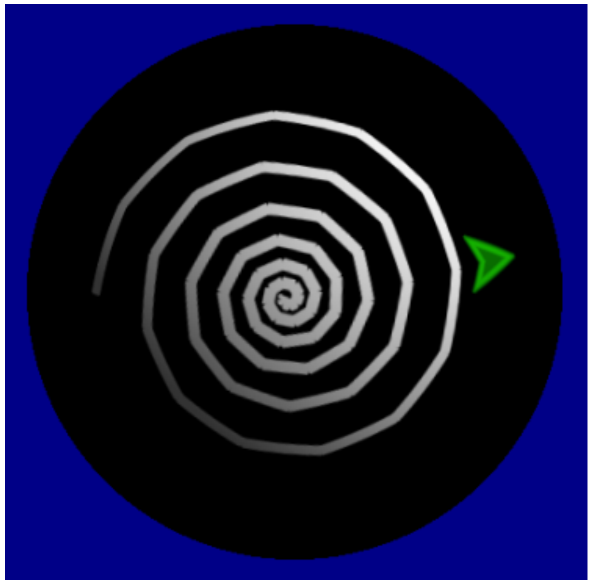

Figure 2. A spiral drawn on the sphere

The above code produces the design shown in Figure 2. (This is the same design we made in Topic 9.) But now let's look at what happens if we set the turtle's turn angle equal to 90 degrees and decrease the amount the turtle walks forward each time, as in the code below:

```
x = 96
a = 90

setheading a

pendown
repeat 12 {
    forward x
    right a
    x = x * 0.8
}


penup
setheading 180
forward 80
```

The above code produces the design shown in Figure 3. Initially, in the outer part of the spiral, we create lines that look like they could form a triangle. But as the spiral moves inward, it starts to create lines that look like they're making squares.

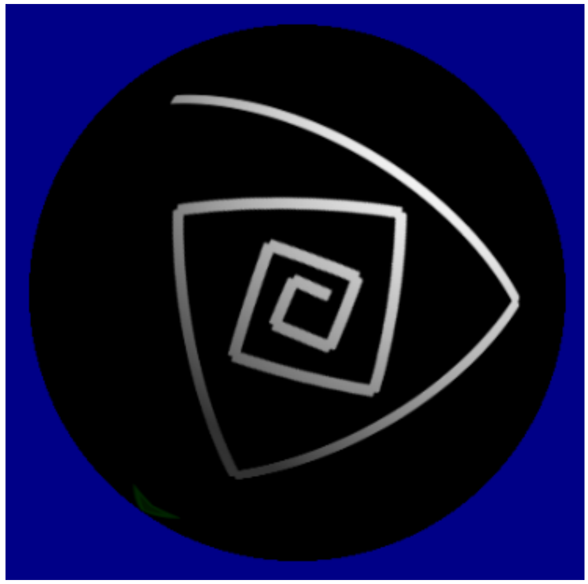

Figure 3. A different spiral

Why does this happen? Well, the angle required to make a square (or any shape) on a sphere depends on how large the square is. (This is why spherical geometry is so weird! When we're dealing with flat planes, the size of the shape doesn't matter!)

So, we can make a small square using 90 degree angles. But we can't make a large square on a sphere using 90 degree angles, because as we discussed earlier, a large *triangle* on a sphere has three 90 degree angles. Thus, on the outside (large) part of the spiral, we make lines that look like sides of a triangle, while in the inner (small) part of the spiral we draw lines that look like sides of a square. This example is a fun way to think about how angles are different on a sphere.

Now that you understand more about how angles work on a sphere, what new designs do you want to make?